CALCULUS AB

SECTION II

Time—1 hour and 15 minutes

Number of problems—5

Percent of total grade—50

SHOW ALL YOUR WORK. INDICATE CLEARLY THE METHODS YOU USE BECAUSE YOU WILL BE GRADED ON THE CORRECTNESS OF YOUR METHODS AS WELL AS ON THE ACCURACY OF YOUR FINAL ANSWERS. IF YOU DO NOT EXPRESS A NUMERICAL ANSWER IN EXACT FORM, FOR EXAMPLE, IN TERMS OF RADICALS, e, OR π, THEN YOU SHOULD EXPRESS THAT ANSWER TO THREE SIGNIFICANT DIGITS. CALCULATORS MAY BE USED ON THIS EXAMINATION.

Notes: (1) In this examination, $\ln x$ denotes the natural logarithm of x (that is, logarithm to the base e). (2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. Let f be the function defined by $f(x) = -2 + \ln(x^2)$.

    (a) For what real numbers x is f defined?

    (b) Find the zeros of f.

    (c) Write an equation for the line tangent to the graph of f at $x = 1$.

2. A particle moves along the X-axis so that at time t its position is given by $x(t) = t^3 - 6t^2 + 9t + 11$.

    (a) What is the velocity of the particle at $t = 0$?

    (b) During what time intervals is the particle moving to the left?

    (c) What is the total distance traveled by the particle from $t = 0$ to $t = 2$?

3. Let f be the function defined for $\frac{\pi}{6} \leqq x \leqq \frac{5\pi}{6}$ by $f(x) = x + \sin^2 x$.

    (a) Find all values of x for which $f'(x) = 1$.

    (b) Find the x-coordinates of all minimum points of f. Justify your answer.

    (c) Find the x-coordinates of all inflection points of f. Justify your answer.
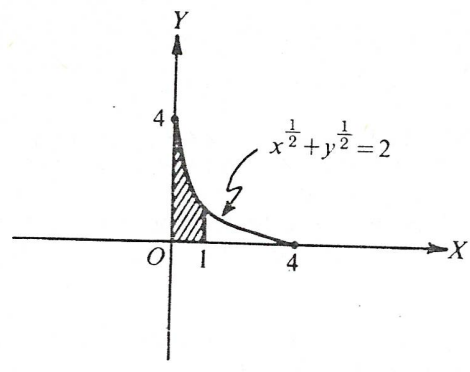
4. The figure above shows the graph of the equation $x^{\frac{1}{2}} + y^{\frac{1}{2}} = 2$. Let R be the shaded region between the graph of $x^{\frac{1}{2}} + y^{\frac{1}{2}} = 2$ and the X-axis from $x = 0$ to $x = 1$.

   (a) Find the area of R by setting up and integrating a definite integral.

   (b) Set up, but <u>do not integrate</u>, an integral expression in terms of a single variable for the volume of the solid formed by revolving the region R about the X-axis.

   (c) Set up, but <u>do not integrate</u>, an integral expression in terms of a single variable for the volume of the solid formed by revolving the region R about the line $x = 1$.

5. At time $t = 0$, a jogger is running at a velocity of 300 meters per minute. The jogger is slowing down with a negative acceleration that is directly proportional to time t. This brings the jogger to a stop in 10 minutes.

   (a) Write an expression for the velocity of the jogger at time t.

   (b) What is the total distance traveled by the jogger in that 10-minute interval?

END OF EXAMINATION